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# Evolution of degree of polarization of partially coherent beams propagation in slant and horizontal atmospheric turbulence

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**Abstract:** In this paper, the analytical expressions for the degree of polarization (DoP) of the electromagnetic Gaussian Schell-model (GSM) beam propagating along a slant path in atmospheric turbulence are obtained. The expressions are used to analyze factors such as the waist radius, amplitude ratio, wavelength, and refractive-index structure constant of the polarization properties of the GSM beam, and we come to some new conclusions. The main results that when the components of the waist radius are equal ( $\sigma_x = \sigma_y$ ), there is a waist radius within  $2 \text{ cm} < \sigma_x < 4 \text{ cm}$  that can provide the most concentrated DoP distribution, and the axis point DoP will eventually approach the initial value as the transmission distance increases, implying that it exhibits self-recovering characteristics. In addition, compared to a horizontal path, the DoP distribution of the GSM beam propagating along a slant path is more concentrated, and the propagation distance corresponding to the maximum axis point DoP is longer. Therefore, when the GSM beam propagates along a slant path, the detector can receive information at longer distances. The research results provide a theoretical support for the control of the polarization state of local beam in coherent optical communication systems.

**Keywords:** Coherent optical communication systems; Polarization control; Atmospheric turbulence

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## 1. Introduction

Polarization is an important property of laser beams and has been extensively studied because of its several important applications in remote sensing, polarization control, and target detection and recognition [1–3]. When laser beams propagate in atmospheric turbulence, the refractive-index random fluctuation affects the polarization properties [4–6]. The study of the polarization properties of laser beams propagating in atmospheric turbulence is thus of great significance.

At present, the theoretical study of polarization properties is based on the cross-spectral density matrix (CSDM) [7–12]. Wolf et al. proposed a unified theory of coherence and polarization, which is of great significance for studying the polarization properties, and derived the expression for the degree of polarization (DoP) and direction of

polarization of a beam propagating in atmospheric turbulence [5, 13–15]; other related studies have discovered that this change is closely related to the light source parameters and turbulence intensity [16]. Zhao et al. obtained a suitable condition for a Gaussian Schell-model (GSM) beam to maintain the state of polarization (SoP). Specifically, the GSM beam maintains the SoP while propagating when three spectral correlation widths ( $\delta_{xx}$ ,  $\delta_{yy}$ , and  $\delta_{xy}$ ) are equal and the beam parameters  $\sigma_x = \sigma_y$  [17]; an experimental study on the modulation of the Stokes parameters confirms this conclusion [18]. Zhao et al. investigated the evolution and modulation in the spectral DoP of a stochastic electromagnetic GSM beam in atmospheric turbulence [19–21]. Cai found that the  $M^2$ -factor of a stochastic electromagnetic GSM beam is primarily related to its initial DoP [22]. Zhang et al. focused on the polarization properties of GSM beams propagating through the anisotropic non-Kolmogorov turbulence of a marine atmosphere [23, 24].

In recent years, several researchers have studied two aspects of polarization properties. One is the change in the

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polarization properties of various beams [25–28], and the other is the change in the polarization properties of a beam propagating along different paths [10–13, 25–29]. Because of the different turbulence mechanisms in the slant path and horizontal path, the polarization properties of the beam are different. Using the unified theory of coherence and polarization, this paper presents the derivation of the expressions for the DoP and compares the difference in polarization properties of a GSM beam propagating through horizontal and slant paths in atmospheric turbulence. Our study is important for controlling the polarization of laser beams in a coherent optical communication system.

## 2. Theory

Based on the unified theory of coherence and polarization [13], a beam is generated by a source located in the  $z = 0$  plane (the source plane) and propagates close to the positive  $z$ -axis into the half-space  $z > 0$ , as shown in Fig. 1. The CSDM is given by [5]

$$\begin{aligned} \vec{W}(\rho_1, \rho_2; z) &\equiv W_{ij}(\rho_1, \rho_2; z) \\ &= \begin{bmatrix} W_{xx}(\rho_1, \rho_2; z) & W_{xy}(\rho_1, \rho_2; z) \\ W_{yx}(\rho_1, \rho_2; z) & W_{yy}(\rho_1, \rho_2; z) \end{bmatrix}, \end{aligned} \quad (1)$$

where  $W_{ij}(\rho_1, \rho_2; z) = \langle E_i(\rho_1; z) E_j^*(\rho_2; z) \rangle$  and  $(i, j = x, y)$ .  $E_i$  and  $E_j$  are respectively the components of the random optical field in the  $x$ - and  $y$ -directions, and the asterisk represents the complex conjugate.  $\rho_1$  and  $\rho_2$  denote two points in the receiver plane at  $z > 0$ , and  $\langle \cdot \rangle$  denotes the ensemble average.

When  $\rho_1 = \rho_2 = \rho$ , by using the generalized Stokes parameters [30], the DoP of the received beam is expressed as

$$P(\rho; z) = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad (2)$$

where  $S_0$ ,  $S_1$ ,  $S_2$ , and  $S_3$  are the Stokes parameters for the random beam and can be expressed by the elements of the CSDM:

$$\begin{aligned} S_0 &= W_{xx}(\rho, \rho; z) + W_{yy}(\rho, \rho; z) \\ S_1 &= W_{xx}(\rho, \rho; z) - W_{yy}(\rho, \rho; z) \\ S_2 &= W_{xy}(\rho, \rho; z) + W_{yx}(\rho, \rho; z) \\ S_3 &= i[W_{yx}(\rho, \rho; z) - W_{xy}(\rho, \rho; z)]. \end{aligned} \quad (3)$$

In the source plane  $z = 0$ , the elements of the CSDM of the GSM source can be expressed by [31, 32]

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2; 0) = A_i A_j B_{ij} \exp \left[ - \left( \frac{r_1^2}{4\sigma_i^2} + \frac{r_2^2}{4\sigma_j^2} \right) \right] \exp \left( - \frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{2\delta_{ij}^2} \right), \quad (4)$$

where  $\mathbf{r}_1$  and  $\mathbf{r}_2$  denote two points in the transverse source plane at  $z = 0$ .  $A_i$  and  $A_j$  are the amplitudes of the GSM beam in the  $x$ - and  $y$ -directions, respectively, and  $\sigma_i$  and  $\sigma_j$  are the waist radii in the  $x$ - and  $y$ -directions, respectively.  $\delta_{ij}$  is the coherence length of the GSM beam, and  $\delta_{ij} = \delta_{ji}$ .  $B_{ij}$  satisfies the following conditions [6]:

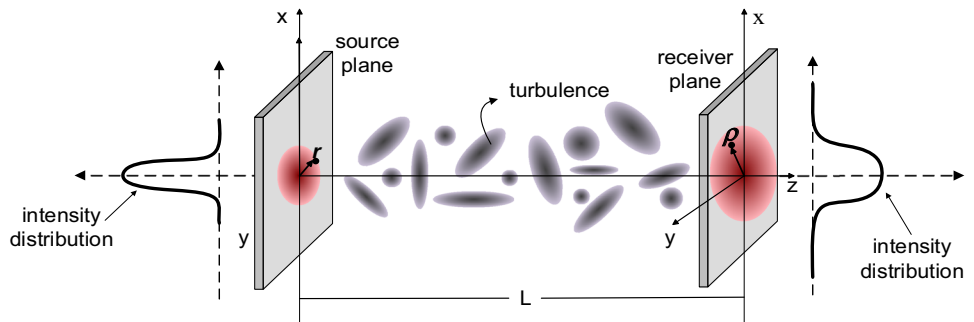
$$B_{ij} = 1 \quad \text{when } i = j; \quad |B_{ij}| \leq 1 \quad \text{when } i \neq j; \quad B_{ij} = B_{ji}^*, \quad (5)$$

According to the extended Huygens–Fresnel principle [28], when the GSM beam propagates from the source plane ( $z = 0$ ) to the receiver plane ( $z = L$ ), the elements of the CSDM is expressed by

$$\begin{aligned} W_{ij}(\rho_1, \rho_2; z) &= A_i A_j B_{ij} \left( \frac{k}{2\pi L} \right)^2 \int d\mathbf{r}_1 \int d\mathbf{r}_2 \\ &\times \exp \left[ - \frac{r_1^2}{4\sigma_i^2} + \frac{r_2^2}{4\sigma_j^2} \right] \exp \left[ - \frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{2\delta_{ij}^2} \right] \\ &\times \exp \left[ - \frac{ik}{2L} (|\rho_1 - \mathbf{r}_1|^2 - |\rho_2 - \mathbf{r}_2|^2) \right] \\ &\times \langle \exp[\psi(\rho_1, \mathbf{r}_1) + \psi^*(\rho_2, \mathbf{r}_2)] \rangle, \end{aligned} \quad (6)$$

where  $L$  is the propagation distance and  $k$  is the optical wave number related to the wavelength  $\lambda$  by  $k = 2\pi/\lambda$ .  $\psi(\rho, \mathbf{r})$  represents the random part of the complex phase of a spherical wave propagating in a turbulent atmosphere from the point  $(\mathbf{r}, 0)$  to  $(\rho, L)$ , and  $\langle \exp[\psi(\rho_1, \mathbf{r}_1) + \psi^*(\rho_2, \mathbf{r}_2)] \rangle$  is written as

**Fig. 1** Beam propagating in atmospheric turbulence



$$\langle \exp[\psi(\rho_1, \mathbf{r}_1) + \psi^*(\rho_2, \mathbf{r}_2)] \rangle = \exp \left[ -\frac{1}{2} D_\psi(\rho_1 - \rho_2, \mathbf{r}_1 - \mathbf{r}_2) \right], \quad (7)$$

where  $D_\psi(\rho_1 - \rho_2, \mathbf{r}_1 - \mathbf{r}_2)$  represents the wave structure function [33]:

$$D_\psi(\rho_1 - \rho_2, \mathbf{r}_1 - \mathbf{r}_2) = 8\pi^2 k^2 \sec \eta \times \int_0^1 \int_0^\infty \kappa \phi_n(\kappa, h) \{1 - J_0[(1 - \xi)(\rho_1 - \rho_2) + \xi(\mathbf{r}_1 - \mathbf{r}_2)|\kappa]\} d\kappa d\xi, \quad (8)$$

where  $J_0(x)$  is the Bessel function of the first kind and zeroth order,  $\eta$  is the zenith angle,  $\kappa$  is the scale of turbulence, and  $\phi_n(\kappa, h)$  is the spectrum of the refractive-index fluctuations. However,  $J_0(x)$  is approximated by the first two terms in the power series expansion [34],  $J_0(x) = 1 - x^2/4$ , thus, Eq. (8) can be simplified as

$$D_\psi(\rho_1 - \rho_2, \mathbf{r}_1 - \mathbf{r}_2) = 2\pi^2 k^2 \sec \eta \int_0^1 \int_0^\infty \kappa^3 \phi_n(\kappa, h) \times \left\{ \left[ (1 - \xi)^2 (\rho_1 - \rho_2)^2 + 2(1 - \xi)\xi(\rho_1 - \rho_2)(\mathbf{r}_1 - \mathbf{r}_2) + \xi^2 (\mathbf{r}_1 - \mathbf{r}_2)^2 \right] \right\} d\kappa d\xi, \quad (9)$$

Command parameters  $M_1$ ,  $M_2$ ,  $M_3$  are respectively represented as

$$\begin{cases} M_1 = 2\pi^2 k^2 \sec \eta \int_0^1 \int_0^\infty \phi_n(\kappa, h) (1 - \xi)^2 \kappa^3 d\kappa d\xi \\ M_2 = 4\pi^2 k^2 \sec \eta \int_0^1 \int_0^\infty \phi_n(\kappa, h) (1 - \xi)\xi \kappa^3 d\kappa d\xi \\ M_3 = 2\pi^2 k^2 \sec \eta \int_0^1 \int_0^\infty \phi_n(\kappa, h) \xi^2 \kappa^3 d\kappa d\xi \end{cases} \quad (10)$$

then, Eq. (9) is simplified as

$$D_\psi(\rho_1 - \rho_2, \mathbf{r}_1 - \mathbf{r}_2) = M_1(\rho_1 - \rho_2)^2 + M_2(\rho_1 - \rho_2) \cdot (\mathbf{r}_1 - \mathbf{r}_2) + M_3(\mathbf{r}_1 - \mathbf{r}_2)^2, \quad (11)$$

The slant path includes the uplink and downlink paths. When the beam propagates along the uplink path,  $\xi = 1 - (h - h_0)/(H - h_0)$ , where  $H$  is the height of the receiver and  $h_0$  is the height of the optics source. In the condition of the downlink path,  $\xi = (h - h_0)/(H - h_0)$ , where  $H$  is the height of the optics source and  $h_0$  is the height of the receiver. If the beam propagates along the horizontal path, Eq. (10) dictates that  $M_3 = 2/3\pi^2 k^2 z \int_0^\infty \kappa^3 \phi_n(\kappa, h = 0) d\kappa$  and  $M_1 = M_2 = 0$ . In this study, we focused on the uplink path and horizontal path. We analyzed the inner- and outer-scale effects through the use of the modified von Karman spectrum model [33]:

$$\phi_n(\kappa, h) = 0.033 C_n^2(h) \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{11/6}}, \quad (12)$$

where  $\kappa_m = 5.92/l_0$ ,  $\kappa_0 \approx 2\pi/L_0$ , and  $l_0$  and  $L_0$  represent the inner scale and outer scale, respectively.  $C_n^2(h)$  is the refractive-index structure constant. The ITU-R model is described as [33]

$$C_n^2(h) = 8.148 \times 10^{-56} v_{\text{RMS}}^2 h^{10} \exp\left(-\frac{h}{1000}\right) + 2.7 \times 10^{-16} \exp\left(-\frac{h}{1500}\right) + C_0 \exp\left(-\frac{h}{100}\right). \quad (13)$$

where  $v_{\text{RMS}} = (v_g^2 + 30.69v_g + 348.91)^{1/2}$  is the wind speed along the vertical path,  $v_g$  is the ground wind speed, and  $C_0$  is the nominal value of the ground level (typically  $1.7 \times 10^{-14} \text{ m}^{-2/3}$ ).

Substituting Eqs. (11) and (7) into Eq. (6) and using  $\rho_1 = \rho_2 = \rho$ , the expressions for the CSDM of the beam in the receiver plane  $z > 0$  is obtained by

$$W_{ij}(\rho, \rho; z) = A_i A_j B_{ij} \left( \frac{k}{2\pi z} \right)^2 \int d\mathbf{r}_c \int d\mathbf{r}_d \times \exp \left[ -\frac{1}{\alpha_{ij}^2} \mathbf{r}_c^2 + \frac{1}{\beta_{ij}} \mathbf{r}_c \mathbf{r}_d + \frac{1}{4\alpha_{ij}^2} \mathbf{r}_d^2 \right] \times \exp \left( -\frac{\mathbf{r}_d^2}{2\delta_{ij}^2} \right) \exp \left[ \frac{ik}{L} (\mathbf{r}_c \mathbf{r}_d - \rho \mathbf{r}_d) \right] \times \exp[-M_3 \mathbf{r}_d^2], \quad (14)$$

where  $r_c = 1/2(r_1 + r_2)$ ,  $r_d = r_1 - r_2$ ,  $1/\alpha_{ij}^2 = 1/(4\sigma_i^2) + 1/(4\sigma_j^2)$  and  $1/\beta_{ij}^2 = 1/(4\sigma_i^2) - 1/(4\sigma_j^2)$ . The variables  $\mathbf{r}_c$  and  $\mathbf{r}_d$  are sequentially integrated; namely, Eq. (14) reduces to

$$W_{ij}(\rho, \rho; z) = \frac{A_i A_j B_{ij}}{\Delta_{ij}^2} \exp \left[ -\frac{\rho^2}{\Delta_{ij}^2 \cdot \alpha_{ij}^2} \right], \quad (15)$$

$$\Delta_{ij}^2 = 1 - \frac{2i}{k\beta_{ij}} z + \frac{1}{k^2} \left[ \frac{1}{\chi_{ij}^2} + M_3 \right] \frac{4}{\alpha_{ij}^2} - \frac{1}{\beta_{ij}^2} z^2, \quad (16)$$

$$\frac{1}{\chi_{ij}^2} = \frac{1}{4\alpha_{ij}^2} + \frac{1}{2\delta_{ij}^2}. \quad (17)$$

From Eqs. (3) and (15), the Stokes parameters of the beam in the receiver plane  $z > 0$  is expressed by



$$\begin{aligned}
S_0 &= \frac{A_x^2}{\Delta_{xx}^2} \exp\left(-\frac{\rho^2}{\Delta_{xx}^2 \sigma_{xx}^2}\right) + \frac{A_y^2}{\Delta_{yy}^2} \exp\left(-\frac{\rho^2}{\Delta_{yy}^2 \sigma_{yy}^2}\right) \\
S_1 &= \frac{A_x^2}{\Delta_{xx}^2} \exp\left(-\frac{\rho^2}{\Delta_{xx}^2 \sigma_{xx}^2}\right) - \frac{A_y^2}{\Delta_{yy}^2} \exp\left(-\frac{\rho^2}{\Delta_{yy}^2 \sigma_{yy}^2}\right) \\
S_2 &= \frac{A_x A_y B_{xy}}{\Delta_{xy}^2} \exp\left(-\frac{\rho^2}{\Delta_{xy}^2 \sigma_{xy}^2}\right) + \frac{A_y A_x B_{yx}}{\Delta_{yx}^2} \exp\left(-\frac{\rho^2}{\Delta_{yx}^2 \sigma_{yx}^2}\right) \\
S_3 &= i \left[ \frac{A_x A_y B_{yx}}{\Delta_{yx}^2} \exp\left(-\frac{\rho^2}{\Delta_{yx}^2 \sigma_{yx}^2}\right) - \frac{A_y A_x B_{xy}}{\Delta_{xy}^2} \exp\left(-\frac{\rho^2}{\Delta_{xy}^2 \sigma_{xy}^2}\right) \right].
\end{aligned} \quad (18)$$

From Eqs. (2) and (17), the axis point DoP in the receiver plane  $z > 0$  is given by

$$P(\rho = 0; z) = \frac{\sqrt{\left(\frac{A_x^2 \Delta_{yy}^2}{A_y^2 \Delta_{xx}^2} - 1\right)^2 + 4|B_{xy}|^2 \frac{A_x^2 \Delta_{yy}^4}{A_y^2 \Delta_{xy}^2 \Delta_{yx}^2}}}{\frac{A_x^2 \Delta_{yy}^2}{A_y^2 \Delta_{xx}^2} + 1}, \quad (19)$$

where  $A_x/A_y = I_x/I_y$  is the ratio of the intensity in the source plane. As can be seen from Eq. (19), the axis point DoP of the beam in the receiver plane is related to  $A_x/A_y$  and is not directly related to the single  $A_x$  and  $A_y$ .

### 3. Results and discussion

According to Eqs. (2), (18), and (19), we will analyze the influence of the waist radius, amplitude ratio, wavelength, and refractive-index structure constant on the polarization properties of the beam in the receiver plane. In the following numerical calculations,  $B_{ij} = 0.5\exp(i\pi/4)$ ,  $\lambda = 1.06 \mu\text{m}$ ,  $\delta_{xx} = 0.02 \text{ m}$ ,  $\delta_{yy} = 0.01 \text{ m}$ ,  $\sigma_x = \sigma_y = 0.01 \text{ m}$ ,  $A_x/A_y = 1$ ,  $\delta_{xy} = \delta_{yx} = 0.025 \text{ m}$ ,  $C_2 n = 1.7 \times 10^{-14} \text{ m}^{-2/3}$ ,  $z = 10 \text{ km}$ , and  $\sigma = \pi/3$ . The coherent length is satisfied by [35]

$$\max\{\delta_{xx}, \delta_{yy}\} \leq \delta_{xy} \leq \min\left\{\delta_{xx}/|B_{xy}|^{0.5}, \delta_{yy}/|B_{xy}|^{0.5}\right\}. \quad (20)$$

#### 3.1. Influence of waist radius

Figure 2 presents the DoP distribution of the receiver beam when the waist radius of the source beam is different in the  $x$ - and  $y$ -directions, and Fig. 4 shows the variation in the axis point DoP of the receiver beam with propagation distance.

Figure 2(a)–(c) presents the DoP distribution of the GSM beam propagating along slant and horizontal paths when  $z = 10 \text{ km}$ . It can be seen that the DoP distribution of the GSM beam propagating along the slant path is more concentrated than that along the horizontal path. Because of the difference in the turbulence term  $M_3$  between the slant and horizontal paths, in Eq. (10), the turbulence term  $M_3$  of the slant path is smaller than that of the horizontal

path. In addition, the DoP of the GSM beam propagating along the slant path is close to 1 when the off-axis distance is relatively small. Comparing Fig. 2(a), (b), and (c), when  $\sigma_x = \sigma_y$ , the DoP distribution first concentrates and then diffuses with the increase in waist radius. There is a waist radius within  $2 \text{ cm} < \sigma_x < 4 \text{ cm}$  that can provide the most concentrated DoP distribution. That is, this waist radius makes the DoP of the receiver beam close to a constant value of 1 when the off-axis distance is the smallest. In order to more effectively prove this phenomenon, we calculate the off-axis distance corresponding to the DoP equal to 1, as shown in Fig. 3. Therefore, these conclusions can be more effectively obtained using Fig. 3. In addition, the DoP distribution is more concentrated when  $\sigma_x < \sigma_y$ , whereas it is more diffuse when  $\sigma_x > \sigma_y$ .

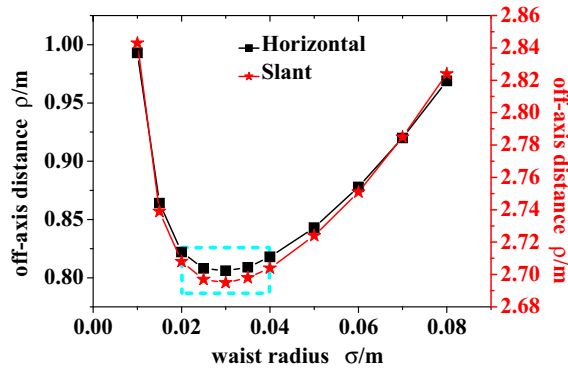
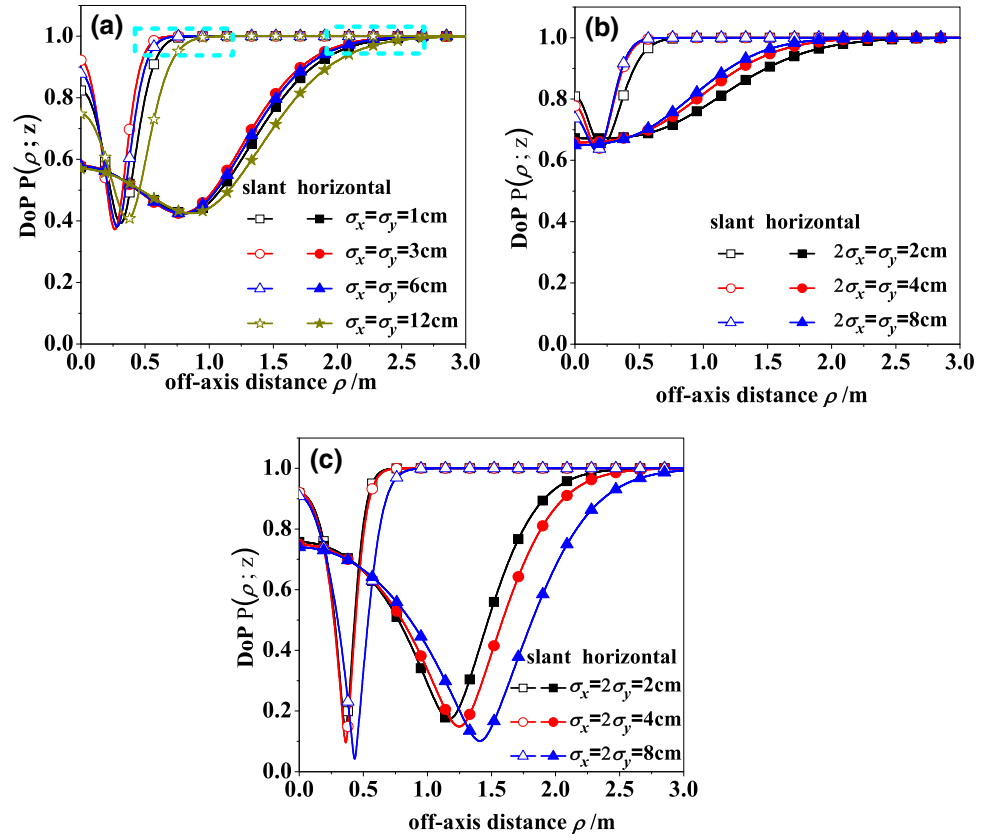
For different  $\sigma_x$  and  $\sigma_y$ , the changes in the axis point DoP of the GSM beam propagating along a slant path and horizontal path with propagation distance are shown in Fig. 4. The results indicate that the propagation distance corresponding to the maximum axis point DoP of the GSM beam propagating along a slant path is longer than that along a horizontal path. The propagation distance corresponding to the maximum axis point DoP will increase with the waist radius. Comparing Fig. 4(a), (b), and (c), it can be seen that when  $\sigma_x = \sigma_y$ , the axis point DoP tends toward the initial value after propagating over a long distance, implying that it exhibits self-recovery characteristics as the propagation distance increases. By contrast, when  $\sigma_x \neq \sigma_y$ , the axis point DoP tends toward a constant value that differs from the initial value. This is because the waist radius of the source beam is different in the  $x$ - and  $y$ -directions, and the influences of turbulence are different.

#### 3.2. Influence of amplitude ratio

Figure 5 presents the DoP distribution of the receiver beam when the amplitude ratio is different in the  $x$ - and  $y$ -directions, and Fig. 6 shows the variation in the axis point DoP of the receiver beam with propagation distance.

Figure 5(a)–(c) presents the DoP distributions of a GSM beam propagating along a slant path and horizontal path, where  $A_x/A_y$  take different values. The DoP distribution is similar to the result in Fig. 2, where the DoP distribution of the GSM beam propagating along a slant path is more concentrated than that along a horizontal path. Comparing Fig. 5(a), (b), and (c), it can be seen that the range of change of the DoP distribution is small when  $A_x/A_y < 1$ , and that of the DoP distribution is relatively large when  $A_x/A_y > 1$ . However, the DoP distribution is more concentrated as the amplitude ratio decreases. Specifically, the smaller the amplitude ratio, the more quickly the DoP reaches 1 when the smaller the off-axis distance.

**Fig. 2** DoP distributions  $P(\rho; z)$  for different waist radii  $\sigma_x$  and  $\sigma_y$ , (a)  $\sigma_x = \sigma_y$ ; (b)  $\sigma_x < \sigma_y$ ; (c)  $\sigma_x > \sigma_y$



**Fig. 3** Off-axis distance  $\rho$  corresponding to the DoP equal to 1 when  $\sigma_x = \sigma_y$

Figure 6(a)–(c) shows the change in the DoP of a GSM beam propagating along a slant path and horizontal path with propagation distance, where  $A_x/A_y$  take different values. The conclusion is similar to the result in Fig. 4. The propagation distance corresponding to the maximum axis point DoP of the GSM beam propagating along a slant path is longer than that propagating along a horizontal path. Comparing Fig. 6(a), (b), and (c), the range of change of the axis point DoP is maximum with the increase in propagation distance when  $A_x/A_y = 1$ . The axis point DoP will increase in  $z = 0$  with the decrease in amplitude ratio

when  $A_x/A_y < 1$ ; however, the reverse is the case when  $A_x/A_y > 1$ .

### 3.3. Influence of wavelength

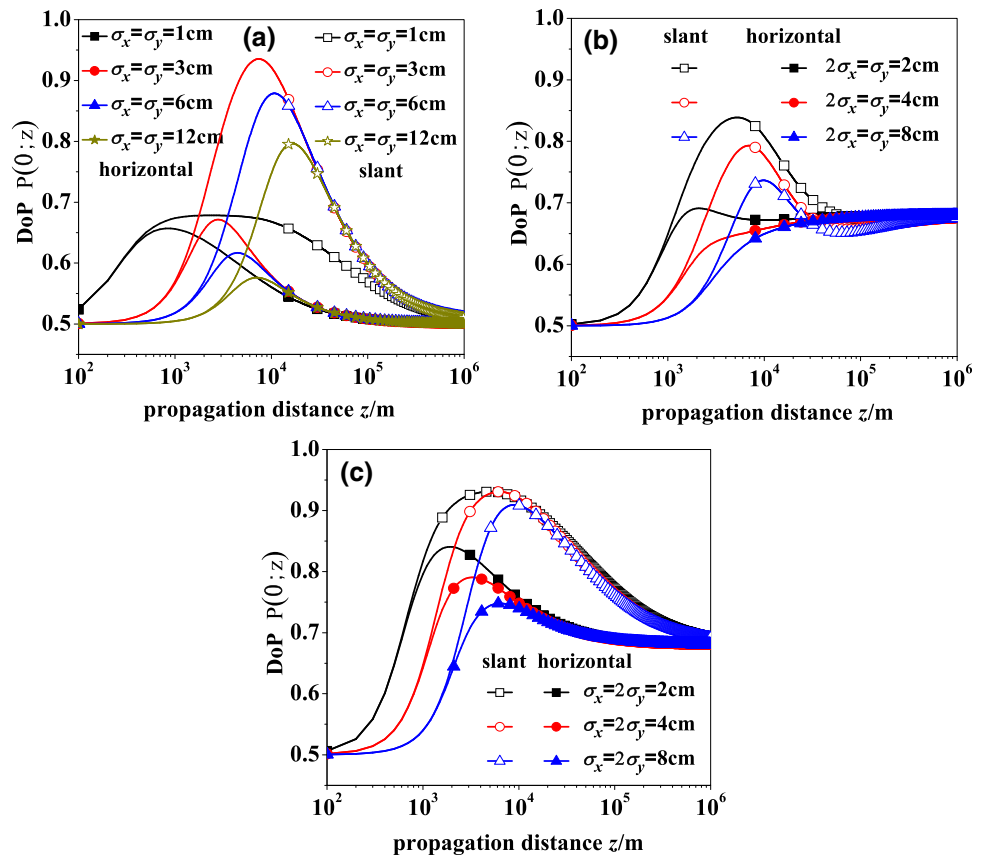
Figure 7 presents the DoP distribution of the receiver beam when the wavelength is different, and Fig. 8 shows the variation in the axis point DoP of the receiver beam with propagation distance.

It can be concluded from Fig. 7 that the DoP distribution of the GSM beam propagating along a slant path is more concentrated than that along a horizontal path under the condition of the same wavelength. And, the longer the wavelength, the more concentrated the DoP distribution of the receiver beam. It can be seen from Fig. 8 that under the condition of the same wavelength, compared to a horizontal path, the detector can receive the information from a farther distance when the GSM beam propagates along a slant path. In addition, the longer the wavelength, the larger the range of change of the axis point DoP.

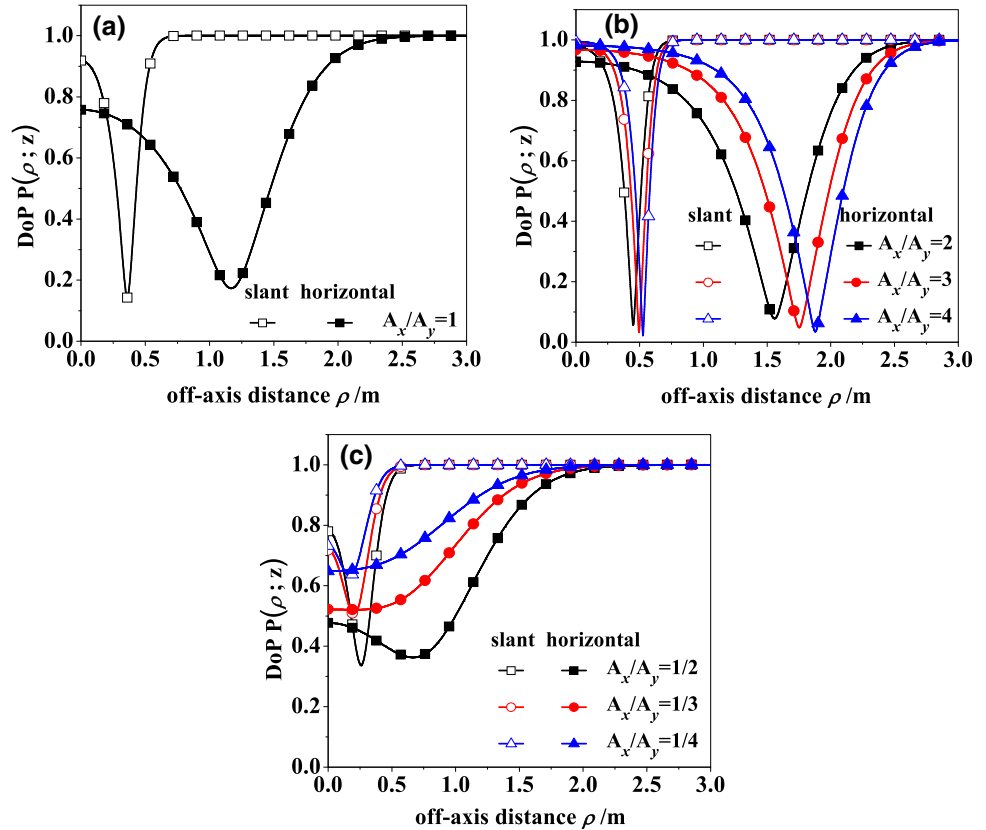
### 3.4. Influence of refractive-index structure constant

Figure 9 presents the DoP distribution of the receiver beam when the refractive-index structure constant is different,

**Fig. 4** Axis point DoP  $P(0; z)$  with the change in propagation distance  $z$  for different waist radii  $\sigma_x$  and  $\sigma_y$ , (a)  $\sigma_x = \sigma_y$ ; (b)  $\sigma_x < \sigma_y$ ; (c)  $\sigma_x > \sigma_y$

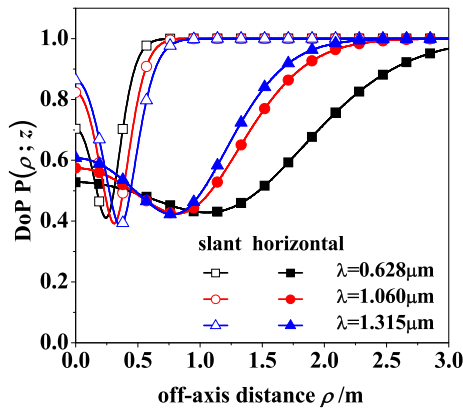
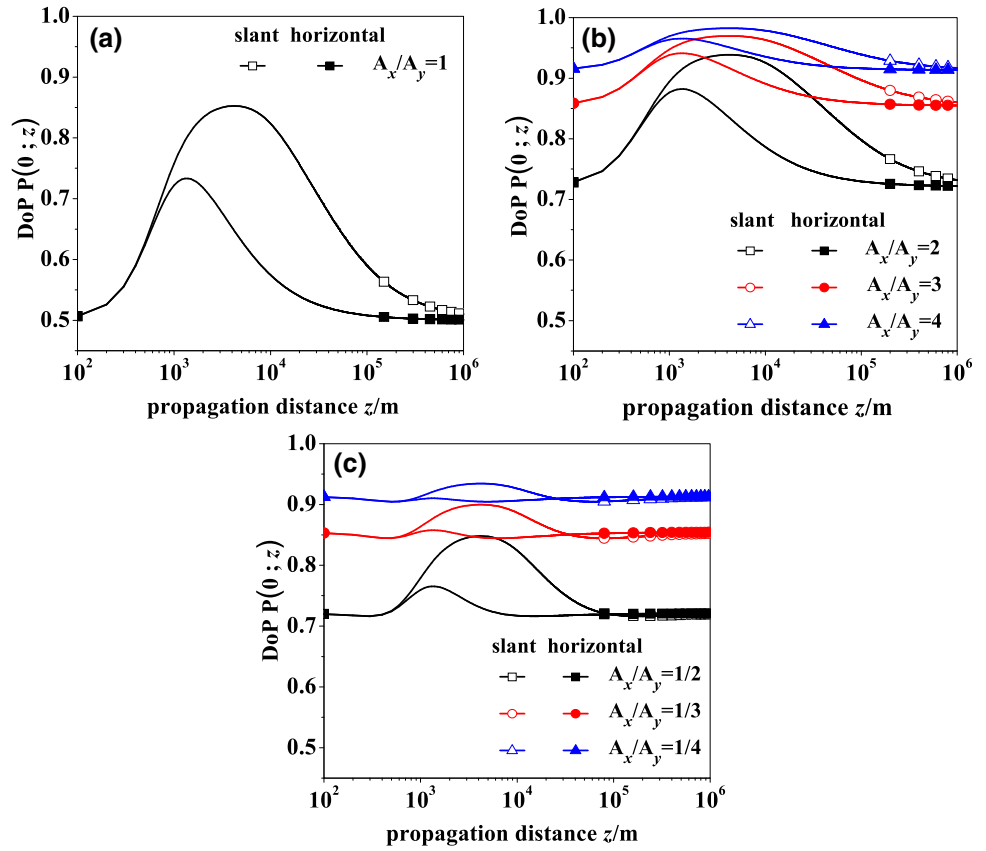


**Fig. 5** DoP distributions  $P(\rho; z)$  for different amplitude ratios  $A_x/A_y$ , (a)  $A_x/A_y = 1$ ; (b)  $A_x/A_y > 1$ ; (c)  $A_x/A_y < 1$





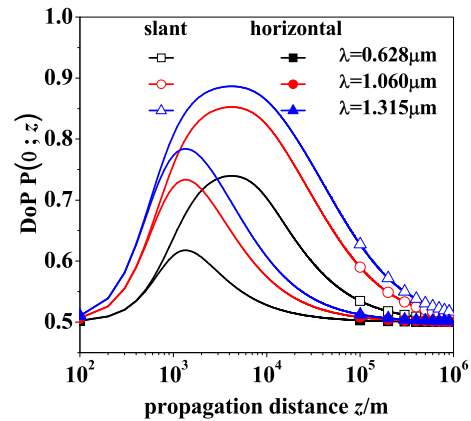
**Fig. 6** Axis point DoP  $P(0; z)$  with the change in propagation distance  $z$  for different amplitude ratios  $A_x/A_y$ , (a)  $A_x/A_y = 1$ ; (b)  $A_x/A_y > 1$ ; (c)  $A_x/A_y < 1$



**Fig. 7** DoP distribution  $P(\rho; z)$  for different wavelengths  $\lambda$

and Fig. 10 shows the variation in the axis point DoP of the receiver beam with propagation.

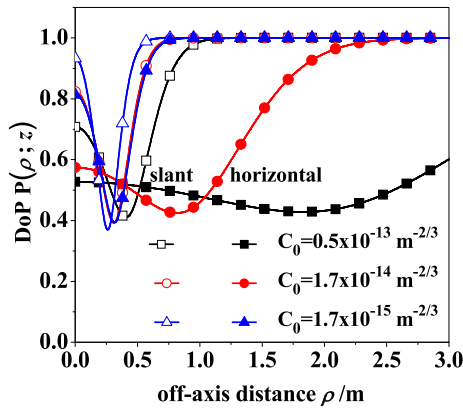
In Fig. 9, the DoP distribution of the GSM beam propagating along a slant path is more concentrated than that along a horizontal path. And, the smaller the refractive-index structure constant, the more concentrated the DoP distribution of the receiver beam. It can be seen from Fig. 10 that under the condition of the same  $C_0$ , the propagation distance corresponding to the maximum axis point DoP of the GSM propagating along the slant path is longer than that along the horizontal path. Compared to the



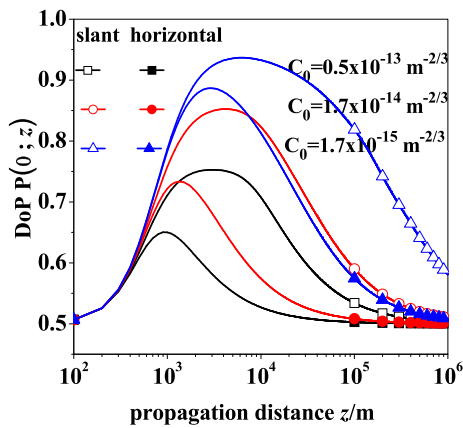
**Fig. 8** Axis point DoP  $P(0; z)$  with the change in propagation distance  $z$  for different wavelengths  $\lambda$

horizontal path, the detector can receive information at farther distances when the GSM beam propagates along a slant path. In addition, the smaller the refractive-index structure constant, the larger the range of change of the axis point DoP.

In a coherent optical communication system, the inconsistency of the polarization states of the signal light and the local oscillator directly affects the reception sensitivity of the entire system, and the advantages of the coherent optical communication system are weakened.



**Fig. 9** DoP distribution  $P(\rho; z)$  for different refractive-index structure constants  $C_0$



**Fig. 10** Axis point DoP  $P(0; z)$  with the change in propagation distance  $z$  for different refractive-index structure constants  $C_0$

With reference to Figs. 4, 6, 8, and 10, it can be seen that the axis point DoP of the beam propagating over a long distance tends to be constant in the atmospheric turbulence, that is, in a coherent optical system, when the communication distance reaches a certain value, the polarization characteristics of the beam are not affected by the atmospheric turbulence, and the polarization states of the signal light and the local oscillator light can be easily controlled to be consistent.

#### 4. Conclusion

In this study, we analyzed the influence of the light source parameters and atmospheric turbulence on the polarization properties of a partially coherent GSM beam when propagating along a slant path and horizontal path. When  $\sigma_x = \sigma_y$ , there is a waist radius within  $2 \text{ cm} < \sigma_x < 4 \text{ cm}$  that can provide the most concentrated DoP distribution, and the axis point DoP will eventually approach the initial value, implying that it exhibits self-recovery characteristics

as the propagation distance increases. Under the conditions of the same light source parameter, the DoP distribution of the GSM beam propagating along the slant path is more concentrated than that along the horizontal path. When the GSM beam propagates along the slant path, the propagation distance corresponding to the maximum axis point DoP is larger; therefore, the detector can receive information at farther distances. Our study can be used as a reference for the selection of light sources and passing information when the GSM beam propagates in atmospheric turbulence. At the same time, it provides an important reference value for the polarization control of atmospheric optical communication systems. Considering the conclusions obtained in this paper regarding the change of the polarization state of a light beam with the propagation distance, future research on coherent optical communication systems should be directed toward understanding how to control the polarization state of local light.

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