Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 6874693, 7 pages
https://doi.org/10.1155/2019/6874693

*Research Article*

# Wind Turbine Clustering Algorithm of Large Offshore Wind Farms considering Wake Effects

**Binbin Zhang and Jun Liu**

*School of Automation and Information Engineering, Xi'an University of Technology, Xi'an 710048, China*

Correspondence should be addressed to Binbin Zhang; zbb521zbb@126.com

Received 22 May 2019; Revised 25 July 2019; Accepted 4 August 2019; Published 9 September 2019

Guest Editor: Miguel Cañas



This paper proposed the SVD (singular value decomposition) clustering algorithm to cluster wind turbines into some group for a large offshore wind farm, in order to reduce the high-dimensional problem in wind farm power control and numerical simulation. Firstly, wind farm wake relationship matrixes are established considering the wake effect in an offshore wind farm, and the SVD of wake relationship matrixes is used to cluster wind turbines into some groups by the fuzzy clustering algorithm. At last, the Horns Rev offshore wind farm is analyzed to test the clustering algorithm, and the clustering result and the power simulation show the effectiveness and feasibility of the proposed clustering strategy.

## 1. Introduction

Wind energy is renewable energy, and it can solve a shortage of fossil fuel and an environmental pollution problem. All wind turbines that will be installed by the end of 2020 can cover close to 9% of the global electricity demand [1]. Offshore wind farm is a new trend because of less planning restriction and better wind condition. Compared with the onshore wind farm, the electrical power production of offshore wind farms is higher and more stable.

There are tens or even hundreds of wind turbines in an offshore wind farm, and they bring a "dimension cruise" challenge [2] for a wind farm control [3–5], numerical simulation [6], and so on. In order to reduce the computation complexity, the common method is to establish an equivalent model for wind farm model reduction [7], and it is a key to cluster the same-feature wind turbines into a group and an equivalent single machine. In recent years, several wind turbine clustering algorithms have been proposed [8–14]. A model reduction method is proposed by a set of orthogonal modes from CFD (computational fluid dynamics) simulation [8]; however, the simulation time is too long for several wind turbines. An aggregated wind farm model is proposed by the average wind speed [9, 10]. A wind turbine clustering algorithm is considered by Hankel singular values [11] or selective modal analysis [12]. However, the wind speed at the downstream wind turbines is smaller than that at the upstream wind turbines in wind farms; this phenomenon is defined as wake effects, and these wind turbine clustering algorithms [9–12] are not considered wake effects of an offshore wind farm.

Coordinates of wind turbines are very regular in an offshore wind farm, and the wind speed and direction are stable, so wake effects of every wind turbine are very regular. Based on the wind farm wake model, wind turbines can be clustered into several groups [13, 14]. The support vector clustering technique is used to cluster wind turbines based on the wind farm layout and incoming wind direction [13]. The $k$-means clustering algorithm divides wind turbines into several groups [14]. However, the wind farm wake model is a high-dimensional mathematical model, and the $k$-means clustering and the support vector clustering algorithms are inefficient and easily converted to a local minimum with more dimensions; at the same time, the results of two clustering algorithms are poor robustness [15]. To solve the high-dimensional problem of wind turbine clustering, SVD (singular value decomposition) is an effective clustering algorithm for large datasets [15].

In this paper, the SVD clustering algorithm is proposed for an offshore wind farm to overcome the high-dimensional problem. A wind farm model is firstly established based on the Jensen wake model, layout of wind farm, and incoming wind speed, a wake combination matrix of every wind turbine is built from a wind farm wake model, and wind turbines are clustered into some groups by an SVD of the wake relationship matrix. At last, an order reduction wind farm model is obtained for power maximizing, power balance control, and so on.

This paper is organized as follows: Section 2 introduces the wind turbine model and the wake model of an offshore wind farm. Then, the SVD clustering algorithm is discussed for the wake model in Section 3. The Horns Rev offshore wind farm is tested in Section 4. Finally, conclusions are drawn in Section 5.

## 2. Wind Farm Wake Model

There are many wake-effect models, such as the Frandsen analytical model [16], Jensen model [17], Larsen model, and CFD (computational fluid dynamics) model [18]. In this paper, the Jensen wake model [18] is adopted because it is simple and suitable for engineering applications [18].

The Jensen wake model is based on the global momentum conservation and assumption of a linear expansion of the wake. Figure 1 shows the basic Jensen model, the radius of the wind turbine is $r_0$, the ambient wind speed is $v_0$, and the wake decay constant is $k$. If a wind turbine is not affected by any upstream wind turbine, $k = 0.04$; otherwise, $k = 0.08$ [19]. $r$ is the radius of the expanding wake, and it can be calculated by (1). And the wind speed $v_1$ inside the wake area at a distance $x$ from the single wind turbine can be calculated by (2), where $C_T$ is the wind turbine thrust coefficient:

$$r = r_0 + kx, \tag{1}$$

$$v_1 = v_0 + v_0\left(\sqrt{1 - C_T} - 1\right)\left(\frac{r_0}{r}\right)^2. \tag{2}$$

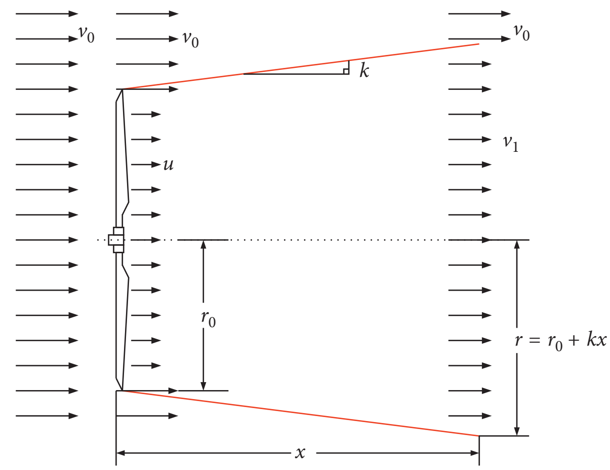



Figure 1: The Jensen wake model [18].

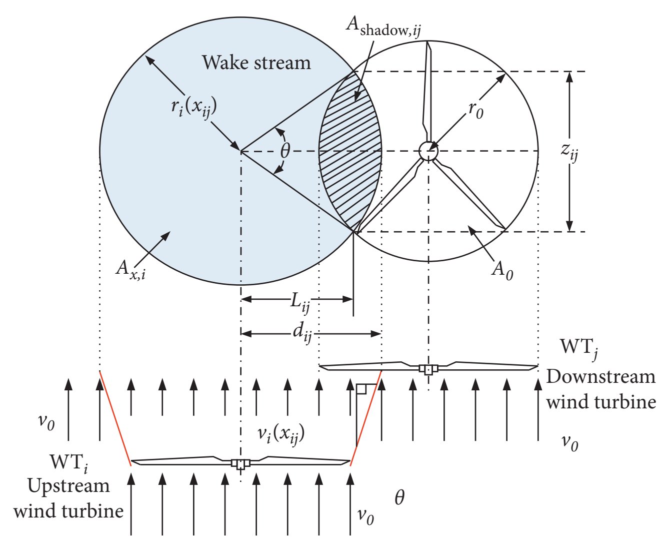



Figure 2: Wind turbine wake shadow [18].

In an offshore wind farm, a downstream wind turbine is affected by multiple wind turbines, and multiple wake effects can be combined into a single wake effect. And the combining multiple wake effects consider the shadowed areas of the upstream wind turbines. The shadow condition, between an upstream wind turbine and a downstream wind turbine, is complete shadowing, quasicomplete shadowing, partial shadowing, and no shadowing. The partial shadowing is shown in Figure 2, the wind turbines' radius $r_0$ is the same, and the swept area of the wind turbine is $A_0$. Then, the shadow area between the two wind turbines can be calculated by

$$A_{\text{shadow},ij} = \left[r_i\left(x_{ij}\right)\right]^2 \cos^{-1}\left(\frac{L_{ij}}{r_i\left(x_{ij}\right)}\right) + r_0^2 \cos^{-1}\left(\frac{d_{ij} - L_{ij}}{r_i\left(x_{ij}\right)}\right) - d_{ij} z_{ij}, \tag{3}$$

where $x_{ij}$ is the distance between the upstream wind turbine $i$ and the downstream wind turbine $j$ along the wind direction and $r_i(x_{ij})$ is the wake stream radius, which can be calculated by (1).

Based on the law of momentum conservation, the combining multiple wake model [19] of the $j$th wind turbine is calculated by

$$v_j = v_0\left[1 - \sum_{i=1}^{n}\left[\left(1 - \sqrt{1 - C_{T,i}}\,\beta_{ij}\right)\right]^2\right], \tag{4}$$

where $\beta_{ij} = (r_0/r_i(x_{ij}))(A_{\text{shadow},ij}/A_0)$.

## 3. A Wind Turbine Clustering Algorithm via SVD

The layout of an offshore wind farm is regular, the distance between turbines is the same, the wake effects of some downstream wind turbines are the same, so the same-wake-effect wind turbines can be clustered as a group and equate a rescaled single wind turbine. From (4), the wind speed of downstream wind turbines is determined by the geographical location and the work condition of upstream wind turbines, and the $C_T$ can be regulated by a wind turbine. Hence, the geographical location is selected as a clustering index [13, 14]. However, the clustering index is 1D data in [13, 14], and the dimension is high as the number of wind turbines increases. A 2D wake relationship matrix can be established from 1D data by analyzing (2), and it is more suitable than 1D data for an offshore wind farm and contains the relative location of wind turbines [20] The 2D wake relationship matrix is a sparse matrix. And the SVD clustering method is effective to solve the high-dimensional sparse matrix clustering problem [21].

*3.1. Estimation of the Wake Relationship Matrix of Every Wind Turbine.* An offshore wind farm has $m$ rows with $n$ wind turbines, and the distance of wind turbines is regular. A wake relationship matrix $A_{ij} \in R^{m\times n}$ of the $i$th row and $j$th column wind turbine is defined as

$$A_{ij} = \left(a_{pq}\right) = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}, \tag{5}$$

where $a_{pq}$ is the element of a wake relationship.

From the wind direction and the wind turbine geographical location, the wake effect between two wind turbines can be obtained. If there is a wake effect between the $ij$th wind turbine and the $pq$th wind turbine, an element of a wake relationship is $\beta_{ij}$; otherwise, the element is 0, if there is not a wake effect, or itself. So the $a_{pq}$ of a wake relationship matrix is defined as

$$a_{pq} = \begin{cases} \left(\dfrac{r_0}{r_i\left(x_{ij}\right)}\right)\left(\dfrac{A_{\text{shadow},ij}}{A_0}\right), & \text{shadowing}, \\ 0, & i = p,\ j = q, \\ 0, & \text{no shadowing}. \end{cases} \tag{6}$$

Generally, the shadowing condition of wind turbines can be judged using the basic geometrical relationship.

*3.2. A SVD Clustering Algorithm of Offshore Wind Farm.* The SVD is an orthogonal matrix reduction, the nonzero singular values contain the most information of the matrix, and it has the advantages of dimension reduction, insensitivity to matrix perturbation, scale invariance of singular values, rotation invariance of singular values, ability to solving the best approximation matrix, and so on [19]. And the proposed wind turbine clustering algorithm flow chart is shown in Figure 3 and is implemented as follows:

Step 1: every wind turbine coordinate, wind direction, and wind turbine parameters, such as the radius of the wind turbine and distance between wind turbines, are obtained.

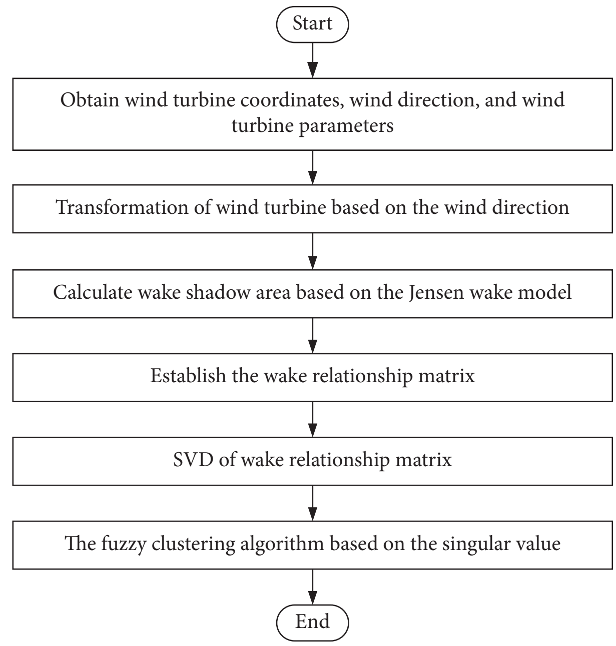



Figure 3: Flow chart of the wind turbine SVD clustering algorithm.

Step 2: an original coordinate $(X, Y)$ of every wind turbine is transformed into another coordinate system $(x, y)$ in the wind direction as (7), where $\theta$ is the wind direction with the positive $X$-axis:

$$\begin{cases} x = X\cos\theta - Y\sin\theta, \\ y = X\sin\theta + Y\cos\theta. \end{cases} \tag{7}$$

Step 3: the wake stream radius and shadow area of the wind farm are calculated based on Section 2.

Step 4: the wake relationship matrix $A_{ij}$ is established from (5) and (6).

Step 5: the singular value decomposition of $A_{ij}$ is calculated as follows:

$$\left[U, S_{ij}, V\right] = \operatorname{svd}\left(A_{ij}\right), \tag{8}$$

where $U$ and $V$ are the left and right singular orthogonal vectors, respectively, and $S_{ij} = \operatorname{diag}(\sigma_1, \ldots, \sigma_p)$, where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_p$ [18].

Step 6: the $S_{ij}$ values are clustered by the fuzzy clustering method [15], and these wind turbines can be clustered into $k$ groups $\{g_1, g_2, \ldots, g_k\}$. And other parameters of the wind turbine are aggregated by a mechanical torque compensation factor method [9]. Finally, the simplified wind farm model is built.

## 4. Case Study

The Horns Rev offshore wind farm in Denmark [22] is used to test this clustering algorithm. It consists of eighty 2 MW wind turbines, and every wind turbine has a hub height $H = 70$ m and a rotor diameter $D = 80$ m. And the wind farm layout is parallelogram columns, the distance between two

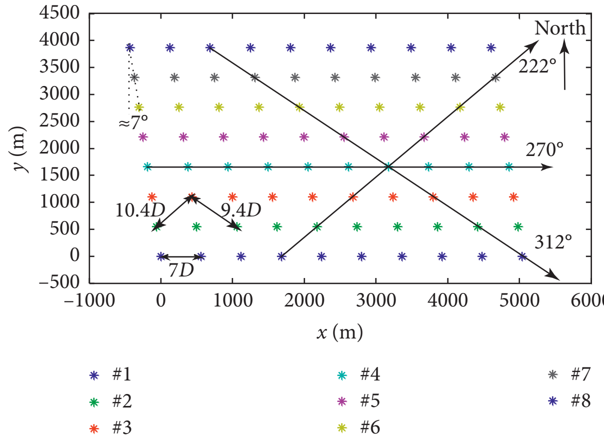



Figure 4: The Horns Rev wind farm layout.

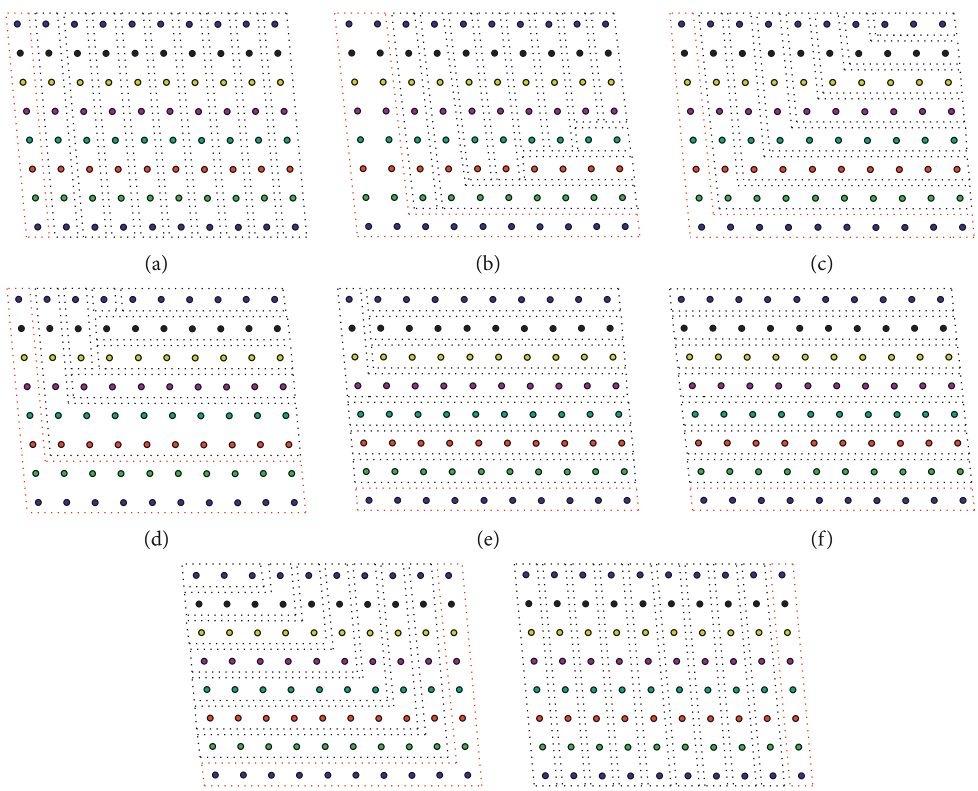


Figure 5: Clustering results of the wind farm at different wind directions: (a) 270°; (b) 246°; (c) 222°; (d) 201°; (e) 180°; (f) 173°; (g) 138°; (h) 90°.

columns is $7D$, the distance between turbines is $7D$, $9.4D$, and $10.4D$ for 0°, 48°, and 312°, respectively, and the angle between the first column and $y$-axis is approximately 7°. Its shape is shown in Figure 4, and it has 8 rows and 11 columns. The wake model of the wind farm is established under eight wind directions which are 270°, 246°, 222°, 201°, 180°, 173°, 138°, and 90° based on the wind farm layout. The clustering results are shown in Figure 5. When the wind direction is 270°, the first-column wind turbines are not affected by other wind turbines, their wind speeds are the ambient wind speed, and wind speeds of other-column wind turbines decrease in turn. And when wind directions are 222° and 312°, the clustering results are similar to the layout of the wind farm. With the wind direction increases, the clustering results are very regular, so a wind farm clustering lookup table can be built for wind farm control and numerical simulation.

In order to verify the clustering results, suppose that the $C_T$ of all wind turbines is the same and $C_T = 0.865$ and the ambient wind speed is 12 m/s. The wind speed of each wind

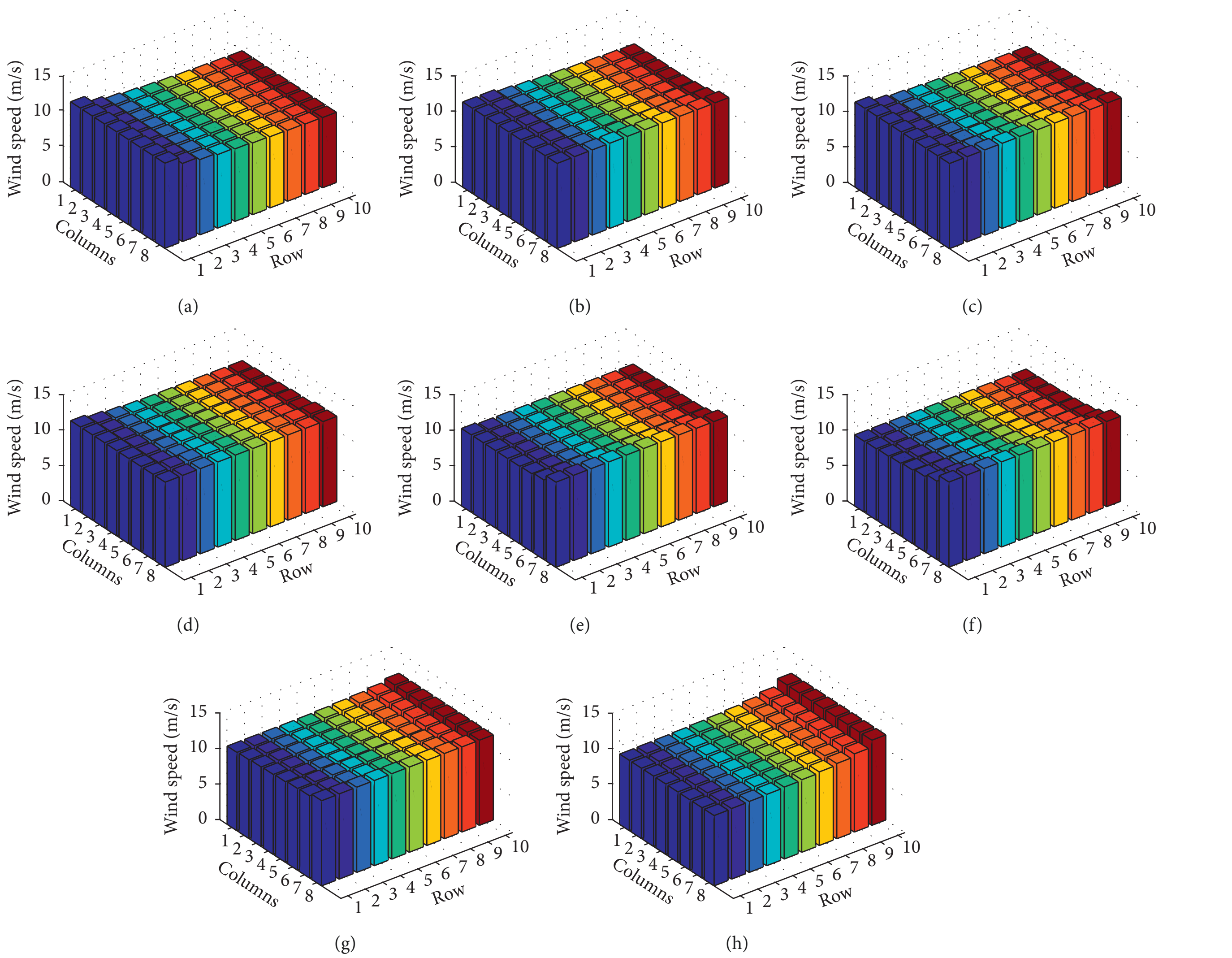



FIGURE 6: Wind speed of each wind turbine: (a) 270°; (b) 246°; (c) 222°; (d) 201°; (e) 180°; (f) 173°; (g) 138°; (h) 90°.

turbine is shown in Figure 6. The wind speed of wind turbines is the same if they are in the same group. From Figure 6, it can be seen that the clustering results are effective and feasible.

The Horns Rev offshore wind farm power simulation is tested by the SVD clustering algorithm and detailed model in MATLAB, which considers every wind turbine powerout. And the power simulations are run on a 3.6 GHz Core i7-4790 CPU with 8 GB RAM using MATLAB version R2014a.

Suppose that the wind speed is 12 m/s and all wind turbines are maximizing power point tracking. And the detailed and equivalent wind farm power curves are shown in Figure 7 at the wind direction range of 180°~270°. From Figure 7, it can be seen that the error between the equivalent model and the detailed model is negligible, and the maximum error is 0.108 MW.

However, when the wind speed of wind farms is over the rated speed, the results of the proposed clustering algorithm may be imprecise. When the ambient wind speed is 17 m/s, it is over the rated wind speed, some wind turbines are power limit controllers, and the $C_T$ of them is different with the

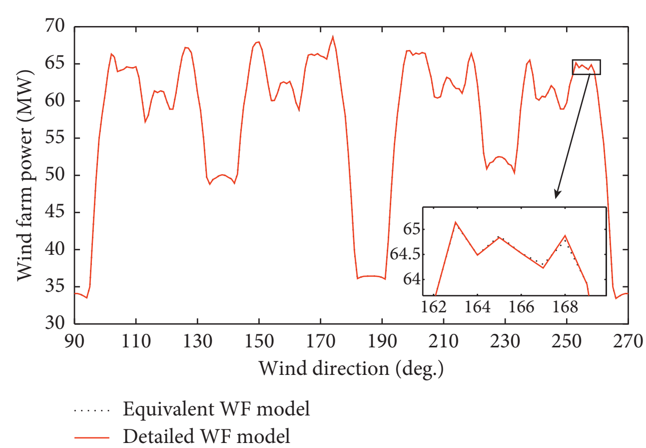



FIGURE 7: Wind farm powerout at different wind directions under 12 m/s.

MPPT wind turbines. And the detailed and equivalent wind farm power curves are shown in Figure 8. From Figure 8, it can be seen that the maximum error is 9.98 MW, and the

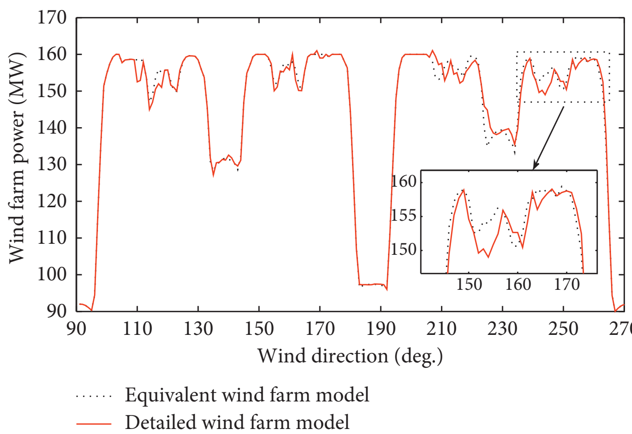



Figure 8: Wind farm powerout at different wind directions under 17 m/s.

Table 1: Computational cost of two wind farm models.

| | Equivalent model | Detailed model |
|---|---|---|
| Computation time (ms) | 2.860 | 6.111 |

error may be large in some wind farm power simulation. So the proposed algorithm can be used when the ambient wind speed is less than the rated speed and the $C_T$ of the same-group wind turbines is the same.

And the computational cost of two wind farm models is shown in Table 1, and the computational efficiency of the proposed wind farm model is higher than that of the detailed model. Moreover, the SVD clustering algorithm is also used for the wind farm power control and power grid simulation considering wind farm, wind farm power-maximizing control, etc.

## 5. Conclusion

The main contribution of this paper is the proposed SVD-based clustering method for large-scale offshore wind farms to solve the high-dimensional problem. Wind turbines can be clustered into several groups based on the location of each wind turbine and wind direction, and the same-group wind turbines, whose $C_T$ is the same, can be equivalent to a single wind turbine, in order to solve the high-dimensional problem in the wind farm control algorithm and numerical simulation.

Based on the layout of wind farm and wind direction, a wind farm wake model is established, a wake relationship matrix is based on the wake model, a singular matrix is calculated by SVD, and finally, wind turbines can be clustered into groups by the fuzzy-means method from singular values. SVD can reduce the high dimension of the wind farm wake model, and the clustering results are relative wind direction and are very regular. Moreover, the large wind farm power control or power grid power simulation with wind farms can reduce the computation time by clustering wind turbines into some groups using this clustering algorithm.

## Data Availability

Previously reported wind turbine coordinates and the Horn Rev wind farm parameters data were used to support this study and are available at DOI: https://doi.org/10.1016/j.renene.2014.06.019. These prior studies and datasets are cited at relevant places within the text as reference [22].

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

This work was supported in part by the Key R&D Project, Shaanxi, China, under Grant no. 2017GY-061.

## References

[1] Wind Power Capacity Worldwide Reaches 600 GW, 53.9 GW Added in 2018, https://wwindea.org/.

[2] T. R. Ayodele, A. Jimoh, J. L. Munda, and A. J. Tehile, "Challenges of grid integration of wind power on power system grid integrity: a review," *International Journal of Renewable Energy Research*, vol. 2, no. 4, pp. 618–626, 2012.

[3] B. Zhang, M. Soltani, W. Hu, P. Hou, Q. Huang, and Z. Chen, "Optimized power dispatch in wind farms for power maximizing considering fatigue loads," *IEEE Transactions on Sustainable Energy*, vol. 9, no. 2, pp. 862–871, 2018.

[4] H. Zhao, Q. Wu, J. Wang, Z. Liu, M. Shahidehpour, and Y. Xue, "Combined active and reactive power control of wind farms based on model predictive control," *IEEE Transactions on Energy Conversion*, vol. 32, no. 3, pp. 1177–1187, 2017.

[5] D.-Y. Li, P. Li, W.-C. Cai, Y.-D. Song, and H.-J. Chen, "Adaptive fault-tolerant control of wind turbines with guaranteed transient performance considering active power control of wind farms," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 4, pp. 3275–3285, 2018.

[6] D. C. C. Crisostomo, A. A. F. Moura, E. P. Rocha, F. M. Cruz, and A. P. Moura, "Educational software for simulation of power and voltage control in power systems connected with wind farms," *IEEE Latin America Transactions*, vol. 16, no. 6, pp. 1603–1609, 2018.

[7] Y. Ni, C. Li, Z. Du, and G. Zhang, "Model order reduction based dynamic equivalence of a wind farm," *International Journal of Electrical Power & Energy Systems*, vol. 83, pp. 96–103, 2016.

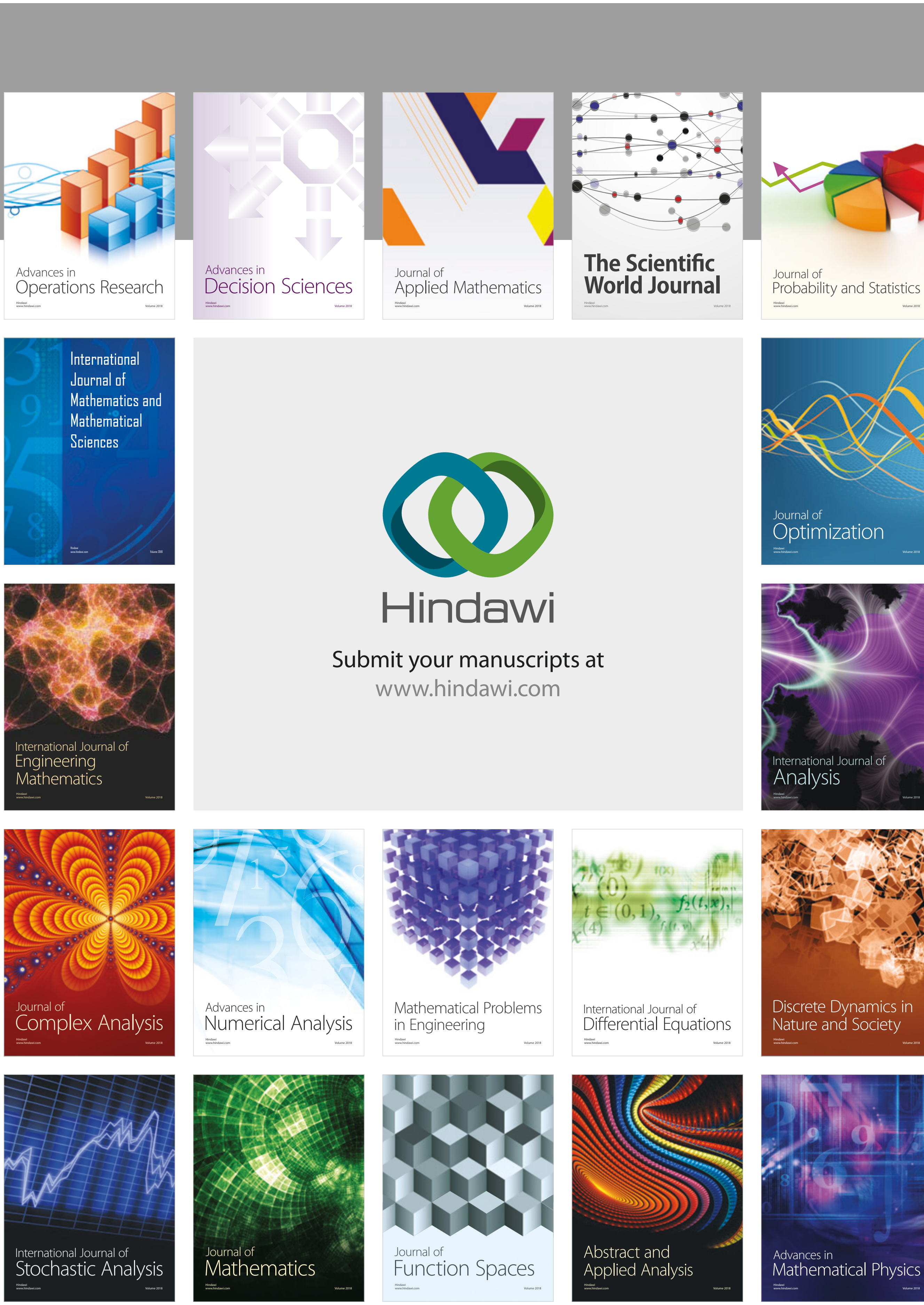
[8] Y. Heggelund, C. Jarvis, and M. Khalil, "A fast reduced order method for assessment of wind farm layouts," *Energy Procedia*, vol. 80, pp. 30–37, 2015.

[9] M. A. Chowdhury, W. X. Shen, N. Hosseinzadeh, and H. R. Pota, "A novel aggregated DFIG wind farm model using mechanical torque compensating factor," *Energy Conversion and Management*, vol. 67, pp. 265–274, 2013.

[10] X. Zha, S. Liao, M. Huang, Z. Yang, and J. Sun, "Dynamic aggregation modeling of grid-connected inverters using Hamilton's-action-based coherent equivalence," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 8, pp. 6437–6448, 2019.

[11] S. Ghosh and N. Senroy, "Balanced truncation based reduced order modeling of wind farm," *International Journal of Electrical Power & Energy Systems*, vol. 53, pp. 649–655, 2013.

[12] H. A. Pulgar-Painemal and P. W. Sauer, “Towards a wind farm reduced-order model,” *Electric Power Systems Research*, vol. 81, no. 8, pp. 1688–1695, 2011.
[13] M. Ali, I.-S. Ilie, J. V. Milanovic, and G. Chicco, “Wind farm model aggregation using probabilistic clustering,” *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 309–316, 2013.
[14] S. Ma, H. Geng, G. Yang, and B. C. Pal, “Clustering-based coordinated control of large-scale wind farm for power system frequency support,” *IEEE Transactions on Sustainable Energy*, vol. 9, no. 4, pp. 1555–1564, 2018.
[15] S. Wierzchoń and M. Kłopotek, *Modern Algorithms of Cluster Analysis*, Springer, Berlin, Germany, 2018.
[16] S. Frandsen, R. Barthelmie, S. Pryor et al., “Analytical modelling of wind speed deficit in large offshore wind farms,” *Wind Energy*, vol. 9, no. 1-2, pp. 39–53, 2006.
[17] N. O. Jensen, *A Note on Wind Generator Interaction*, Risø National Laboratory, Roskilde, Denmark, 1983.
[18] R. Shakoor, M. Y. Hassan, A. Raheem, and Y.-K. Wu, “Wake effect modeling: a review of wind farm layout optimization using Jensen’s model,” *Renewable and Sustainable Energy Reviews*, vol. 58, pp. 1048–1059, 2016.
[19] I. Katic, J. Højstrup, and N. O. Jensen, “A simple model for cluster efficiency,” in *Proceedings of the European Wind Energy Association Conference and Exhibition*, A. Raguzzi, Rome, Italy, October 1987.
[20] P. Drineas, A. Frieze, R. Kannan, S. Vempala, and V. Vinay, “Clustering large graphs via the singular value decomposition,” *Machine Learning*, vol. 56, no. 1–3, pp. 9–33, 2004.
[21] F. González-Longatt, P. Wall, and V. Terzija, “Wake effect in wind farm performance: steady-state and dynamic behavior,” *Renewable Energy*, vol. 39, no. 1, pp. 329–338, 2012.
[22] Y.-T. Wu and F. Porté-Agel, “Modeling turbine wakes and power losses within a wind farm using LES: an application to the horns rev offshore wind farm,” *Renewable Energy*, vol. 75, pp. 945–955, 2015.


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
The Scientific
World Journal
Journal of
Probability and Statistics
International
Journal of
Mathematics and
Mathematical
Sciences
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of
Optimization
International Journal of
Engineering
Mathematics
International Journal of
Analysis
Journal of
Complex Analysis
Advances in
Numerical Analysis
Mathematical Problems
in Engineering
International Journal of
Differential Equations
Discrete Dynamics in
Nature and Society
International Journal of
Stochastic Analysis
Journal of
Mathematics
Journal of
Function Spaces
Abstract and
Applied Analysis
Advances in
Mathematical Physics